Subject A211

2023 Study Guide

Contents

Section 0	Course structure	Page 2
Section 1	Before you start	Page 3
Section 2	Core study material	Page 4
Section 3	Skills	Page 6
Section 4	Syllabus and Core Reading	Page 10

Please note that Subject A211 is based on the first 13 chapters of the Institute and Faculty of Actuaries' (IFoA) Subject CM1. Therefore the majority of your study materials will refer to CM1 and not A211.

0 The course structure

There are two parts to the Subject A211 course. The parts are broken down into chapters.

The following table shows how the parts, the chapters and the syllabus items relate to each other. We have also given you a broad indication of the length of each chapter. This table should help you plan your progress across the study session.

Part	*Ch*	*Title*	*No of pages*	*Syllabus objectives*
1	1	Principles of actuarial modelling	25	1.1
	2	Cashflow models	24	1.2
	3	The time value of money	27	2.1, 2.3
	4	Interest rates	40	2.1
	5	Real and money interest rates	13	2.2
	6	Discounting and accumulating	32	2.4
	7	Level annuities	36	2.5
	8	Increasing annuities	30	2.5
	9	Equations of value	21	3.1
2	10	Loan schedules	31	3.2
	11	Project appraisal	32	3.3
	12	Bonds, equity and property	54	2.2, 3.2
	13	Term structure of interest rates	51	2.6, 2.7

1 Before you start

When studying for the actuarial exams, you will need:

- a copy of the Formulae and Tables for Examinations of the Faculty of Actuaries and the Institute of Actuaries, 2nd Edition (2002) – these are often referred to as simply the Yellow Tables or the Tables
- a 'permitted' scientific calculator – you will find the list of permitted calculators on the profession's website. Please check the list carefully, since it is reviewed each year.

2 Core study material

This section explains the role of the Syllabus, Core Reading and supplementary ActEd text. It also gives guidance on how to use these materials most effectively in order to pass the exam.

Some of the information below is also contained in the introduction to the Core Reading produced by the Institute and Faculty of Actuaries.

Syllabus

The relevant individual Syllabus Objectives are included at the start of each course chapter and a complete copy of the Syllabus is included in this Study Guide. We recommend that you use the Syllabus as an important part of your study.

Core Reading

The Core Reading has been produced by the Institute and Faculty of Actuaries. The purpose of the Core Reading is to ensure that tutors, students and examiners understand the requirements of the syllabus for the qualification examinations for Fellowship of the Institute and Faculty of Actuaries.

The Core Reading supports coverage of the syllabus in helping to ensure that both depth and breadth are re-enforced. It is therefore important that students have a good understanding of the concepts covered by the Core Reading.

The examinations require students to demonstrate their understanding of the concepts given in the syllabus and described in the Core Reading; this will be based on the legislation, professional guidance *etc* that are in force when the Core Reading is published, *ie* on 31 May in the year preceding the examinations.

Therefore the exams in April and September 2023 will be based on the Syllabus and Core Reading as at 31 May 2022. We recommend that you always use the up-to-date Core Reading to prepare for the exams.

Examiners will have this Core Reading when setting the papers. In preparing for examinations, students are advised to work through past examination questions and may find additional tuition helpful. The Core Reading will be updated each year to reflect changes in the syllabus and current practice, and in the interest of clarity.

ActEd text

Core Reading deals with each syllabus objective and covers what is needed to pass the exam. However, the tuition material that has been written by ActEd enhances it by giving examples and further explanation of key points. Here is an excerpt from some ActEd Course Notes to show you how to identify Core Reading and the ActEd material. **Core Reading is shown in this bold font.**

Note that in the example given above, the index *will* fall if the actual share price goes below the theoretical ex-rights share price. Again, this is consistent with what would happen to an underlying portfolio.

After allowing for chain-linking, **the formula for the investment index then becomes:**

$$I(t) = \frac{\sum_i N_{i,t} P_{i,t}}{B(t)}$$

This is ActEd text

This is Core Reading

where $N_{i,t}$ is the number of shares issued for the ith constituent at time t;

$B(t)$ is the base value, or divisor, at time t.

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3 Skills

Technical skills

The Core Reading and exam papers for these subjects tend to be very technical. The exams themselves have many calculation and manipulation questions. The emphasis in the exam will therefore be on *understanding* the mathematical techniques and applying them to various, frequently unfamiliar, situations. It is important to have a feel for what the numerical answer should be by having a deep understanding of the material and by doing reasonableness checks.

As a high level of pure mathematics and statistics is generally required for the Core Principles subjects, it is important that your mathematical skills are extremely good. If you are a little rusty you may wish to consider purchasing additional material to help you get up to speed. The course 'Pure Maths and Statistics for Actuarial Studies' is available from ActEd and it covers the mathematical techniques that are required for the Core Principles subjects, some of which are beyond A-Level (or equivalent) standard. You do not need to work through the whole course in order – you can just refer to it when you need help on a particular topic. An initial assessment to test your mathematical skills and further details regarding the course can be found on our website at **ActEd.co.uk.**

Study skills

Overall study plan

We suggest that you develop a realistic study plan, building in time for relaxation and allowing some time for contingencies. Be aware of busy times at work, when you may not be able to take as much study leave as you would like. Once you have set your plan, be determined to stick to it. You don't have to be too prescriptive at this stage about what precisely you do on each study day. The main thing is to be clear that you will cover all the important activities in an appropriate manner and leave plenty of time for revision and question practice.

Aim to manage your study so as to allow plenty of time for the concepts you meet in these courses to 'bed down' in your mind. Most successful students will probably aim to complete the courses at least a month before the exam, thereby leaving a sufficient amount of time for revision. By finishing the courses as quickly as possible, you will have a much clearer view of the big picture. It will also allow you to structure your revision so that you can concentrate on the important and difficult areas.

You can also try looking at our discussion forum on the internet, which can be accessed at **ActEd.co.uk/forums** (or use the link from our home page at **ActEd.co.uk**). There are some good suggestions from students on how to study.

Study sessions

Only do activities that will increase your chance of passing. Try to avoid including activities for the sake of it and don't spend time reviewing material that you already understand. You will only improve your chances of passing the exam by getting on top of the material that you currently find difficult.

Ideally, each study session should have a specific purpose and be based on a specific task, *eg* '*Finish reading Chapter 3 and attempt Practice Questions 3.4, 3.7 and 3.12'*, as opposed to a specific amount of time, *eg 'Three hours studying the material in Chapter 3*'.

Try to study somewhere quiet and free from distractions (*eg* a library or a desk at home dedicated to study). Find out when you operate at your peak, and endeavour to study at those times of the day. This might be between 8*am* and 10*am* or could be in the evening. Take short breaks during your study to remain focused – it's definitely time for a short break if you find that your brain is tired and that your concentration has started to drift from the information in front of you.

Order of study

We suggest that you work through each of the chapters in turn. To get the maximum benefit from each chapter you should proceed in the following order:

1. Read the Syllabus Objectives. These are set out in the box at the start of each chapter.

2. Read the Chapter Summary at the end of each chapter. This will give you a useful overview of the material that you are about to study and help you to appreciate the context of the ideas that you meet.

3. Study the Course Notes in detail, annotating them and possibly making your own notes. Try the self-assessment questions as you come to them. As you study, pay particular attention to the listing of the Syllabus Objectives and to the Core Reading.

4. Read the Chapter Summary again carefully. If there are any ideas that you can't remember covering in the Course Notes, read the relevant section of the notes again to refresh your memory.

5. Attempt (at least some of) the Practice Questions that appear at the end of the chapter.

It's a fact that people are more likely to remember something if they review it several times. So, do look over the chapters you have studied so far from time to time. It is useful to re-read the Chapter Summaries or to try the Practice Questions again a few days after reading the chapter itself. It's a good idea to annotate the questions with details of when you attempted each one. This makes it easier to ensure that you try all of the questions as part of your revision without repeating any that you got right first time.

Once you've read the relevant part of the notes and tried a selection of questions from the Practice Questions you should attempt the corresponding assignment. It can seem a bit depressing to analyse the errors you made, but you will increase your chances of passing the exam by learning from your mistakes.

To be really prepared for the exam, you should not only know and understand the Core Reading but also be aware of what the examiners will expect. Your revision programme should include plenty of question practice so that you are aware of the typical style, content and marking structure of exam questions. You should attempt as many past exam questions as you can.

Active study

Here are some techniques that may help you to study actively.

1. Don't believe everything you read. Good students tend to question everything that they read. They will ask 'why, how, what for, when?' when confronted with a new concept, and they will apply their own judgement. This contrasts with those who unquestioningly believe what they are told, learn it thoroughly, and reproduce it (unquestioningly?) in response to exam questions.

2. Another useful technique as you read the Course Notes is to think of possible questions that the examiners could ask. This will help you to understand the examiners' point of view and should mean that there are fewer nasty surprises in the exam room. Use the Syllabus to help you make up questions.

3. Annotate your notes with your own ideas and questions. This will make you study more actively and will help when you come to review and revise the material. Do not simply copy out the notes without thinking about the issues.

4. Attempt the questions in the notes as you work through the course. Write down your answer before you refer to the solution.

5. Attempt other questions and assignments on a similar basis, *ie* write down your answer before looking at the solution provided. Attempting the assignments under exam conditions has some particular benefits:
   - It forces you to think and act in a way that is similar to how you will behave in the exam.
   - The knowledge that you are going to do an assignment under exam conditions can act as a powerful incentive to make you study each part as well as possible.
   - It is also quicker than trying to write perfect answers.

You can find further information on how to study in the UK Profession's Student Handbook, which you can download from their website at:

actuaries.org.uk/qualify

Revision and exam skills

Revision skills

You will have sat many exams before and will have mastered the exam and revision techniques that suit you. However it is important to note that due to the high volume of work involved in the Core Principles subjects it is not possible to leave all your revision to the last minute. Students who prepare well in advance have a better chance of passing their exams on the first sitting.

Unprepared students find that they are under time pressure in the exam. Therefore it is important to find ways of maximising your score in the shortest possible time. Part of your preparation should be to practise a large number of exam-style questions under timed exam conditions as soon as possible. This will:

- help you to develop the necessary understanding of the techniques required
- highlight the key topics, which crop up regularly in many different contexts and questions
- help you to practise the specific skills that you will need to pass the exam.

Exam question skill levels

Exam questions are not designed to be of similar difficulty. The Institute and Faculty of Actuaries specifies different skill levels that questions may be set with reference to.

Questions may be set at any skill level:

- Knowledge – demonstration of a detailed knowledge and understanding of the topic
- Application – demonstration of an ability to apply the principles underlying the topic within a given context
- Higher Order – demonstration of an ability to perform deeper analysis and assessment of situations, including forming judgements, taking into account different points of view, comparing and contrasting situations, suggesting possible solutions and actions, and making recommendations.

4 Syllabus and Core Reading

Syllabus

The Syllabus for Subject A211 is given here. To the right of each objective are the chapter numbers in which the objective is covered in this course.

Aim

The aim of the Financial Mathematics subject is to provide a grounding in the principles of modelling as applied to actuarial work – focusing particularly on deterministic models which can be used to model and value known cashflows.

Competences

On the successful completion of this subject, the candidate will be able to

1. Describe the basic principles of actuarial modelling.
2. Describe, interpret and discuss the theories of interest rates.
3. Describe, interpret and discuss mathematical techniques used to model and value known cashflows.

Syllabus topics

1.	The basics of modelling	(15%)
2.	Theory of interest rates	(40%)
3.	Equation of value and its applications	(45%)

The weightings are indicative of the approximate balance of the assessment of this subject between the main syllabus topics, averaged over a number of examination sessions. The weightings also have a correspondence with the amount of learning material underlying each syllabus topic. However, this will also reflect aspects such as:

- the relative complexity of each topic, and hence the amount of explanation and support required for it
- the need to provide thorough foundation understanding on which to build the other objectives
- the extent of prior knowledge which is expected
- the degree to which each topic area is more knowledge or application based.

Skills level

The use of a specific command verb within a syllabus objective does not indicate that this is the only form of question which can be asked on the topic covered by that objective. The Examiners may ask a question on any syllabus topic using any of the agreed command verbs, as are defined in the document "Command verbs used in the Associate and Fellowship written examinations".

Questions may be set at any skill level: Knowledge (demonstration of a detailed knowledge and understanding of the topic), Application (demonstration of an ability to apply the principles underlying the topic within a given context) and Higher Order (demonstration of an ability to perform deeper analysis and assessment of situations, including forming judgements, taking into account different points of view, comparing and contrasting situations, suggesting possible solutions and actions, and making recommendations).

In the Financial Mathematic subject, the approximate split of assessment across the three skill types is 25% Knowledge, 65% Application and 10% Higher Order skills.

Detailed syllabus objectives

1. The basics of modelling

    1.1 Describe the principles of actuarial modelling. (Chapter 1)

        1.1.1 Describe why and how models are used.

        1.1.2 Explain the benefits and limitations of modelling.

        1.1.3 Explain the difference between a stochastic and a deterministic model, and identify the advantages/disadvantages of each.

        1.1.4 Describe the characteristics of, and explain the use of, scenario-based and proxy models.

        1.1.5 Describe, in general terms, how to decide whether a model is suitable for any particular application.

        1.1.6 Explain the difference between the short-run and long-run properties of a model, and how this may be relevant in deciding whether a model is suitable for any particular application.

        1.1.7 Describe, in general terms, how to analyse the potential output from a model, and explain why this is relevant to the choice of model.

        1.1.8 Describe the process of sensitivity testing of assumptions and explain why this forms an important part of the modelling process.

        1.1.9 Explain the factors that must be considered when communicating the results following the application of a model.

1.2 Describe how to use a generalised cashflow model to describe financial transactions. (Chapter 2)

1.2.1 State the inflows and outflows in each future time period and discuss whether the amount or the timing (or both) is fixed or uncertain for a given cashflow process.

1.2.2 Describe in the form of a cashflow model the operation of financial instruments (like a zero-coupon bond, a fixed-interest security, an index-linked security, a current account, cash on deposit, a credit card, an equity, an interest-only loan, a repayment loan and an annuity certain) and an insurance contract (like endowment, term assurance, contingent annuity, car insurance and health cash plans).

2. Theory of interest rates

2.1 Show how interest rates may be expressed in different time periods. (Chapters 3 and 4)

2.1.1 Describe the relationship between the rates of interest and discount over one effective period arithmetically and by general reasoning.

2.1.2 Derive the relationships between the rate of interest payable once per measurement period (effective rate of interest) and the rate of interest payable p (> 1) times per measurement period (nominal rate of interest) and the force of interest.

2.1.3 Calculate the equivalent annual rate of interest implied by the accumulation of a sum of money over a specified period where the force of interest is a function of time.

2.2 Demonstrate a knowledge and understanding of real and money interest rates. (Chapters 5 and 12)

2.3 Describe how to take into account time value of money using the concept of compound interest and discounting. (Chapter 3)

2.3.1 Accumulate a single investment at a constant rate of interest under the operation of simple and compound interest.

2.3.2 Define the present value of a future payment.

2.3.3 Discount a single investment under the operation of a simple (commercial) discount at a constant rate of discount.

2.4 Calculate the present value and accumulated value for a given stream of cashflows under the following individual or combination of scenarios: (Chapter 6)

2.4.1 Cashflows are equal at each time period.

2.4.2 Cashflows vary with time which may or may not be a continuous function of time.

2.4.3 Some of the cashflows are deferred for a period of time.

2.4.4 The rate of interest or discount is constant.

2.4.5 The rate of interest or discount varies with time which may or may not be a continuous function of time.

2.5 Define and derive the following compound interest functions (where payments can be in advance or in arrears) in terms of i, v, n, d, δ, $i^{(p)}$ and $d^{(p)}$: (Chapters 7 and 8)

2.5.1 $a_{\overline{n}|}$, $s_{\overline{n}|}$, $a^{(p)}_{\overline{n}|}$, $s^{(p)}_{\overline{n}|}$, $\ddot{a}_{\overline{n}|}$, $\ddot{s}_{\overline{n}|}$, $\ddot{a}^{(p)}_{\overline{n}|}$, $\ddot{s}^{(p)}_{\overline{n}|}$, $\bar{a}_{\overline{n}|}$ and $\bar{s}_{\overline{n}|}$.

2.5.2 ${}_{m|}a_{\overline{n}|}$, ${}_{m|}a^{(p)}_{\overline{n}|}$, ${}_{m|}\ddot{a}_{\overline{n}|}$, ${}_{m|}\ddot{a}^{(p)}_{\overline{n}|}$ and ${}_{m|}\bar{a}_{\overline{n}|}$.

2.5.3 $(Ia)_{\overline{n}|}$, $(I\ddot{a})_{\overline{n}|}$, $(I\bar{a})_{\overline{n}|}$ and $(\overline{I\bar{a}})_{\overline{n}|}$ and the respective deferred annuities.

2.6 Show an understanding of the term structure of interest rates. (Chapter 13)

2.6.1 Describe the main factors influencing the term structure of interest rates.

2.6.2 Explain what is meant by, derive the relationships between and evaluate:

- discrete spot rates and forward rates.
- continuous spot rates and forward rates.

2.6.3 Explain what is meant by the par yield and yield to maturity.

2.7 Show an understanding of duration, convexity and immunisation of cashflows. (Chapter 13)

2.7.1 Define the duration and convexity of a cashflow sequence, and illustrate how these may be used to estimate the sensitivity of the value of the cashflow sequence to a shift in interest rates.

2.7.2 Evaluate the duration and convexity of a cashflow sequence.

2.7.3 Explain how duration and convexity are used in the (Redington) immunisation of a portfolio of liabilities.

3. Equation of value and its applications

3.1 Define an equation of value. (Chapter 9)

3.1.1 Define an equation of value, where payment or receipt is certain.

3.1.2 Describe how an equation of value can be adjusted to allow for uncertain receipts or payments.

3.1.3 Understand the two conditions required for there to be an exact solution to an equation of value.

3.2 Use the concept of equation of value to solve various practical problems. (Chapters 10 and 12)

3.2.1 Apply the equation of value to loans repaid by regular instalments of interest and capital. Obtain repayments, interest and capital components, the effective interest rate (APR) and construct a schedule of repayments.

3.2.2 Calculate the price of, or yield (nominal or real allowing for inflation) from, a bond (fixed-interest or index-linked) where the investor is subject to deduction of income tax on coupon payments and redemption payments are subject to deduction of capital gains tax.

3.2.3 Calculate the running yield and the redemption yield for the financial instrument as described in 3.2.2.

3.2.4 Calculate the upper and lower bounds for the present value of the financial instrument as described in 3.2.2 when the redemption date can be a single date within a given range at the option of the borrower.

3.2.5 Calculate the present value or yield (nominal or real allowing for inflation) from an ordinary share or property, given constant or variable rate of growth of dividends or rents.

3.3 Show how discounted cashflow and equation of value techniques can be used in project appraisals. (Chapter 11)

3.3.1 Calculate the net present value and accumulated profit of the receipts and payments from an investment project at given rates of interest.

3.3.2 Calculate the internal rate of return, payback period and discounted payback period and discuss their suitability for assessing the suitability of an investment project.

Assessment

Two hour written examination

Core Reading

The Subject A211 Course Notes include the Core Reading in full, integrated throughout the course.

Accreditation

The Institute and Faculty of Actuaries would like to thank the numerous people who have helped in the development of the material contained in the Core Reading.

Further reading

The exam will be based on the relevant Syllabus and Core Reading and the ActEd course material will be the main source of tuition for students.